Università degli Studi di Padova

SCIENZE MATEMATICHE

Course unit English denomination	**General enrichment strategies for finite element methods to solve Poisson problem with Dirichlet boundary conditions**
SS	MATH-05/A
Teacher in charge (if defined)	Nudo Federico
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	November 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	The course will cover the following topics: • Introduction to finite element methods. • Limitations of standard triangular and simplicial linear finite elements. • Overview of enrichment strategies for finite element methods • Conforming and nonconforming enrichment approach. • Enrichment strategies for specific finite elements and error bounds in L^1 and L^∞ norm. • Implementation of enriched finite element methods to solve the Poisson problem with Dirichlet boundary conditions.
Learning goals	By the end of the course, students are expected to: • Understand the fundamental principles of FEM and EFEM. • Learn to apply FEM and EFEM to solve practical engineering problems. • Develop the skills necessary to implement enrichment strategies in numerical simulations.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	This course is designed for students who possess a basic understanding of numerical analysis and it focuses specifically on enrichment strategies for the

	Poisson problem with Dirichlet boundary conditions, although these strategies are applicable to a broader class of elliptic boundary value problems. Familiarity with linear algebra is essential, and basic MATLAB programming skills are recommended for the practical implementation of the discussed methods.
Examination methods (in applicable)	Solving exercises and giving a seminar on a course-related topic
Suggested readings	1. P. G. Ciarlet. The finite element method for elliptic problems, SIAM, 2002. 2. A. Guessab. Sharp Approximations based on Delaunay Triangulations and Voronoi Diagrams, NSU Publishing and Printing Center., 2022. 3. A. J. M. Ferreira. MATLAB Codes for Finite Element Analysis, Springer, 2009. 4. F. Dell'Accio, F. Di Tommaso, A. Guessab, F. Nudo. A general class of enriched methods for the simplicial linear finite elements, Applied Mathematics and Computation, 456:128149, 2023. 5. M. Eskandari-Ghadi, D. Mehdizadeh, A. Morshedifard, M. Rahimian. A family of exponentially gradient elements for numerical computation of singular boundary value problems, Engineering Analysis with Boundary Elements, 80: 184–198, 2017.
Additional information	

UNIVERSITÀ DEGLI STUDI DI PADOVA

SCIENZE MATEMATICHE

Course unit English denomination	**Functional convex ordering of stochastic processes: a constructive approach with applications to Finance**
SS	MATH-03/B, STAT-04/A
Teacher in charge (if defined)	Gilles Pagès
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	May 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Convex ordering: definitions and first (static) examples - Convex ordering(s) for Rd-valued random vectors - Characterization of convex orders - First examples: convex ordering of Gaussian vectors, European vanilla options with convex payoff in a Black-Scholes model, Value-at-Risk and Expected shortfall. - Toward functional order: the case of Asian option. • Functional convex ordering(s): definition and characterization - Propagation of convexity - The case of martingale (and scaled) Brownian diffusions - Application to path-dependent European options convex payoffs in local volatility models - Extension to jump diffusions (SDEs driven by L´evy processes) • From European to American path-dependent options for Brownian and jump diffusions • Convex ordering for McKean-Vlasov SDEs • Application to the comparison of mean-field games (optional) • Convex ordering in a non-Markovian framework: the case of stochastic Volterra equations. • Application to variance swaps in a Quadratic rough Heston stochastic volatility model.

Learning goals	– Familiarize the audience with the different notions of convex order between random variables and their links with the usual risk measures in finance. – Extend these notions to a functional framework in order to apply it to Markovian or non-Markovian stochastic processes. – Analyze the connections between convex order and propagation of convexity by a semigroup associated with various Markov processes. – Different families of processes will be studied: ARCH processes (discrete time), Brownian or jump diffusion processes, solutions of McKean-Vlasov equations, stochastic Volterra processes. – Applications to the sensitivity of path-dependent options to “functional volatility” will be detailed. - Most of the results will be obtained by passing to the limit from the simulable numerical approximation schemes, of the Euler scheme type, which makes it possible to define effective approximation protocols respecting convex ordering and convexity propagation for the calculation of prices of complex optional products having a path-dependent payoff.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Probability Theory, Stochastic processes, stochastic calculus
Examination methods (if applicable)	It can be a standard exam with or without documents or more likely the reading of research papers combined with numerical experiments
Suggested readings	P. Carr, C.-O. Ewald and Y.Xiao, On the qualitative effect of volatility and duration on prices of Asian options, Finance Research Letters, 5(3):162–171, 2008. B. Jourdain, G. Pagès, Convex ordering for stochastic Volterra equations and their Euler schemes, Fin. & Stoch., 29(1):1-62, 2025. B. Jourdain, G. Pagès, Convex ordering of solutions to one-dimensional SDEs, arXiv:2312.09779, 2023. N. El Karoui, M. Jeanblanc & S.E. Shreve, Robustness of the Black and Scholes formula. Math. Financ. 8(2):9–126, 1998. B. Hajek, Mean stochastic comparison of diffusions. Z.Wahrsch. Verw. Gebiete, 68(3):315– 329, 1985. F. Hirsch, B. Roynette, C. Profeta & M. Yor, Peacocks and Associated Martingales, with Explicit Constructions, Springer, 2011. P.-L. Lions, M. Musiela, Convexity of solutions of parabolic equations, C. R. Acad. Sci. Paris, S´er. I 342 (2006) 915–921. G. Pagès, Convex order for path-dependent derivatives: a dynamic programing approach, Séminaire de Probabilités, XLVIII, LNM 2168, Springer, Berlin, 33-96, 2016.

Y. Liu, G.Pagès, Functional convex order for the scaled McKean–Vlasov processes, Ann. Appl. Probab. 33(6A):4491–4527, 2023. DOI: 10.1214/22-AAP1924

Y. Liu, G.Pagès, Monotone convex order for the McKean–Vlasov processes, Stoch. Proc. & their Appli., 152:312-338, 2022, ISSN 0304-4149, https://doi.org/10.1016/j.spa.2022.06.003.

Additional information

Course unit English denomination	**Reflection groups**
SS	MATH-02/A
Teacher in charge (if defined)	Giulio Peruginelli, Andriy Regeta
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	November 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Reflections in a Euclidean space and complex reflections. Reflection groups. Examples. • Coxeter groups and Coxeter graphs. The classification of finite reflection groups. Platonic solids. • Crystallographic groups (Weyl groups). • How do we parametrize orbits? Basics on commutative algebra and invariant theory. Orbit spaces and fundamental regions. • Invariant polynomials. The case of the symmetric group. • Chevalley-Shephard-Todd theorem with examples. • Degrees of a finite reflection group. *(if time permits)* • Basics on affine algebraic varieties. • Platonic solids, finite subgroups of SL2(C) and Kleininan singularities.
Learning goals	The aim of the course is to stress the special nature of groups generated by reflections and the role of symmetry in different situations in mathematics.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

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Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic notions on groups, rings and vector spaces that are usually covered in a bachelor degree in mathematics. If needed, definitions and basic theorems would be recalled along the course.
Examination methods (if applicable)	Solving exercises and/or giving a seminar on a paper related to the content of the course.
Suggested readings	[B] N. Bourbaki, Elements of Mathematics, Chapter IV, Coxeter Groups and Tits systems and Chapter V, Groups Generated by Reflections, Springer, English translation by Andrew Pressley, from the 1968 original version. [H] J. Humphreys, Reflection Groups and Coxeter Groups, Cambridge University Press, 1992. [R] A. Regeta, Lectures on Reflection Groups and Invariant Theory, lecture notes, available at https://andriyregeta.wixsite.com/homepage [S] P. Slodowy, Platonic solids, Kleinian singularities, and Lie groups, in: Proceedings of the Third Midwest Algebraic Geometry Conference held at the University of Michigan, Ann Arbor, USA, November 14-15, 1981 Ed; I Dolgachev. [D] I. Dolgachev, Reflection groups in algebraic geometry, Bull. A.M.S. 45 (2008), 1–60.
Additional information	Symmetry is a crucial concept in mathematics and the first natural examples of symmetries one may think of are reflections. Groups generated by reflections (reflection groups) include well-known families, such as the symmetric groups and the dihedral groups. Reflection groups have very special properties, that can be seen for instance in their group structure, and in the nature of their orbit spaces. Classical corner stones in the theory are the classification of finite reflection groups in a Euclidean space by means of Coxeter graphs -including the classification of Platonic solids- and the Chevalley-Shephard-Todd's theorem. The latter characterizes finite groups generated by complex reflections acting on a linear space as those for which the ring of invariant polynomial functions is a ring of polynomials (i.e., the orbit space is again linear). After getting a grip on these basic facts, the course is intended to move on to the role that reflection groups play in singularity theory.

Course unit English denomination	**Linear Parabolic Equations in Hilbert Spaces: analysis and numerical approximation**
SS	MATH-05/A
Teacher in charge (if defined)	Federico Piazzon
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	May 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	The course consists of two closely related and complementary parts: the first 10 hours will be devoted to the construction of the solutions of certain classes of linear parabolic equations formulated as evolution equations in Hilbert spaces. The second part of the course (6 hours) concerns the numerical approximation of such solutions. First we consider the semi-discrete approximation by the Faedo-Galerkin approach, then we exploit the properties of analytic semigroups to define a fully-discrete sequence of approximations by means of the Laplace transform and its numerical inversion. Due to the time constraint, only the most important (and/or instructive) results will be proven, while many others will be only presented and discussed. Some classical equations (e.g., heat eq., convection-diffusion eq., Sobolev eq., and visco-elastic eq.) will be used as examples both to apply the presented theoretical results and verify their hypothesis, and to test the introduced approximation techniques. **Part 1 (10h - 5 lectures)** 1) Quadratic forms and linear operators on Hilbert spaces; 2) Accretive operators, generation of contraction and analytic semigroups, relation with Laplace transform; 3) Solving first-order (in time) linear non-degenerte explicit parabolic equations; 4) More general linear parabolic equations than ut = Lu + f: implicit and second-order equations; 5) Classical examples: heat, convection-diffusion, visco-elastic, and Sobolev equations. **Part 2 (6h - 3 lectures)**

	1) Galerkin method for elliptic problems, standard error estimates; 2) Semi-discretization by Faedo-Galerkin method, error analysis and convergence; 3) Fully discrete approximation by Laplace transform and quadrature, convergence analysis for sectorial operators.
Learning goals	The course will offer, in the framework of linear parabolic PDEs, the opportunity of running into the whole scientific process of analyzing a mathematical problem, constructing its solution, and developing a robust numerical approximation method by exploiting the same properties of the problem that surfaced in the analysis step.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Real and functional analysis, basics of numerical analysis. All the essential notions will be briefly recalled during the lectures.
Examination methods (if applicable)	Either oral examination on the content of the course, or presentation of a related research paper. Whenener the background of the student includes some programming skills, the presentation of numerical experiments might be included in the exam.
Suggested readings	[1] R. E. Showalter. Hilbert space methods for partial differential equations. Monographs and Studies in Mathematics, Vol. 1. Pitman, London-San Francisco, Calif.-Melbourne, 1977. [2] Alexandre Ern and Jean-Luc Guermond. Finite elements III first-order and time-dependent PDEs, volume 74 of Texts in Applied Mathematics. Springer, Cham, [2021] c2021. [3] Vidar Thom´ee. Galerkin finite element methods for parabolic problems, volume 25 of Springer Series in Computational Mathematics. Springer-Verlag, Berlin, second edition, 2006. [4] E. Brian Davies. Linear operators and their spectra, volume 106 of Cambridge Studies in Advanced Mathematics. Cambridge University Press, Cambridge, 2007.
Additional information	

Course unit English denomination	**Principal Bundles**
SS	MATH-02/B
Teacher in charge (if defined)	Oren Ben-Bassat
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	October 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	● Short introduction to Grothendieck topologies and sheaves. E´ tale cohomology, Cˇ ech cohomology. Examples. ● Principal bundles and torsors in topology, arithmetic geometry, complex analytic geometry, differential geometry, and algebraic geometry. ● Groupoids, moduli spaces of vector bundles, vector bundles on the projective line and other algebraic curves. ● Stable bundles, Higgs bundles, Hitchin systems and their quantization. ● Topological Quantum Field Theories and Frobenius algebras. ● Defining Topological Quantum Field Theories with G-bundles. **Optional topics**: - Related topics in representation theory and group cohomology
Learning goals	The course provides an introduction to the theory of principal bundles and shows some applications in different fields of mathematics, ranging from arithmetic to mathematical physics.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	It will be helpful to come with some familiarity with topics like commutative algebra, homological algebra, vector bundles, differential geometry, algebraic geometry, different types of cohomology. We will try to fill in the necessary category theory as we go along.
Examination methods (if applicable)	Oral presentation of an argument related to the topics presented during the lectures.
Suggested readings	- “Hitchin systems and their quantization”, Pavel Etingof, Henry Liu, https://arxiv.org/abs/2409.09505 - “Frobenius algebras and 2D topological quantum field theories” (short version), Joachim Kock https://mat.uab.cat/ kock/TQFT/FS.pdf - “Vector Bundles and K-theory”, by Allen Hatcher https://pi.math.cornell.edu/ hatcher/VBKT/VB.pdf
Additional information	

Course unit English denomination	**Random Graphs and Networks**
SS	MATH-03/B
Teacher in charge (if defined)	Gianbattista Giacomin
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	February 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Complex networks have captured the attention of the scientific community in recent years due to their prevalence in a wide variety of real-world scenarios, such as social networks, biological systems, and technological infrastructures. These networks exhibit largescale behaviors that reveal common properties, notably the "small-world" effect and the "scalefree" phenomenon. Random graphs serve as mathematical models that facilitate the analysis of these large-scale features. Roughly, random graphs can be described as random variables taking values on a set of graphs, hence well suited to capture both probabilistic and combinatorial aspects of the real-world networks. The course will focus on different classes of random graphs. We will start from the definition of the Erdos-Rényi random graph, one of simplest model one could think of. Despite its simplicity, this model presents relevant and unforeseen large-scale features that will be discussed along the course, including an interesting phase transition related to presence of a giant connected component. Keeping in mind the properties of real networks, we will then introduce and discuss three different families of random graphs: The Inhomogeneous Random Graph, the Configuration Model and the Preferential Attachment Model. Each model captures different features of real-world networks, such as heavy-tailed degree distributions and small world behavior, while maintaining mathematical tractability. The course will conclude discussing more applied topics such as generative models for community detection in networks. In particular, the Stochastic Block Model (SBM) and its connection with belief propagation and the reconstruction problem on trees will be presented as time permits. Lectures (tentative schedule)

	1. Basic setting: graphs, trees, random graph setting, and main properties of the real-world networks. 2. Erdös-Rényi (ER) random graphs: Uniform and Binomial model; monotonicity and thresholds. 3. ER random graphs structure: trees containment, Poisson paradigm, largest component, connectivity. 4. Exploration process of a graph and a random walk perspective. Tool: Branching Processes. 5. Emergence of a giant component in ER- random graphs. 6. Inhomogeneous random graphs (IRG): degree sequence and scale-free property. 7. Configuration Model (CM): construction and simplicity probability. Uniform random graphs. 8. Phase transition and small world phenomenon in the IRG and in the CM. Tool: Multi-type branching process. 9. Preferential Attachment Model (PAM): construction, scale free and small world properties. 10. Perspectives I: Community structure and community detection. 11. Perspectives II: Stochastic Block Model (SBM) and reconstruction on trees. 12. Problem for solution: the spectrum of random graphs.
Learning goals	Random graph theory sits at the intersection of probability, combinatorics, and graph theory, offering elegant and rigorous methods to study complex networks. These methods provide insights into both real-world phenomena and abstract mathematical structures. The aim of this mini-course is to equip PhD students with a rigorous understanding of both the theoretical and applied aspects of random graphs as models for complex networks. Upon completion, students are expected: • to achieve a solid understanding of fundamental concepts in random graph theory, including classical random graph models and properties relevant to real-world networks; • to be able to implement the main techniques involved in the study of random graphs, including probabilistic methods, combinatorial tools, and analytical methods; • to gain insights into advanced topics, such as problems in community detection or characterizing the spectrum of random graphs, and to develop the skills necessary to understand scientific papers on these subjects.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No

Prerequisites (not mandatory)	Basic knowledge of probability theory: discrete random variables, finite and countable probability spaces, convergence theorems (law of large number, central limit theorem).
Examination methods (if applicable)	Seminar on a paper
Suggested readings	1. R. van der Hofstad. Random graphs and complex networks. Vol. 1. Cambridge Series in Statistical and Probabilistic Mathematics, [43]. Cambridge University Press, Cambridge, 2017. (available on the author webpage) 2. R. van der Hofstad. Random graphs and complex networks. Vol. 2. Cambridge Series in Statistical and Probabilistic Mathematics, [54]. Cambridge University Press, Cambridge, 2024. (available on the author webpage) 3. A. Frieze, M. Karoński. Introduction to random graphs. Cambridge University Press, Cambridge, 2016. (available on the author webpage) 4. R. van der Hofstad. Stochastic processes on random graphs. Lecture notes for the 47th Summer School in Probability Saint-Flour, 2017. (available on the author webpage) 5. E. Abbe. Community Detection and Stochastic Block Models: Recent Developments. Journal of Machine Learning Research, 18(177), 86 pp, 2017.
Additional information	

Course unit English denomination	**Volterra Equations and their Markovian Lift(s)**
SS	MATH-03/B
Teacher in charge (if defined)	Ofelia Bonesini
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	May 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	***1. Preliminaries on Stochastic Volterra Equations (SVEs)*** • Definition and main features • Comparison with standard SDEs: loss of Markovianity and semimartingality • Existence and uniqueness results (in particular, for the singular kernel case) ***2. Applications in finance: rough volatility models*** • Fractional Brownian motion and its role as the main player in volatility modelling • Rough Bergomi model: definition and its meaning • Connections between SVEs and (multifactor) fractional stochastic volatility models ***3. Markovian lifts*** • Markovian lifts foundation: restoring Markovianity in SVEs • Measure-valued lift: genesis, how this is used in practice, and limitations • Curve-valued lift: link with forward variance curves in finance • Approximation techniques via Markovian representations ***4. SVEs and Path-dependent PDEs*** • Connections between SVEs, PPDEs (path-dependent PDEs), and BSPDEs (backward stochastic PDEs)

	• Integral representations and Laplace transform methods ***5. Advanced topics and open problems*** • Recent research on uniqueness and weak solutions for SVEs • Applications of Itô-type formulas in Volterra processes • Open questions in Markovian approximations and numerical methods
Learning goals	1. **Understand a generalisation of standard SDEs (SVEs) for which** Markovianity and semimartingality do not hold. 2. **Analyse Rough Volatility models to u**nderstand why fractional Brownian motion is particularly suitable for financial modeling. 3. **Explore the infinite dimensional setting established by the use of Markovian lifts**, in particular by comparing the different lifting techniques (measure-valued, curve-valued). Apply lifts to restore Markovianity and help in functional Itô calculus. 4. **Indentify the connection between SVEs and Path-Dependent PDEs by linking the two** to backward stochastic PDEs. 5. Identify the key challenges in the established methods to detect potential further developments and improvements in the established theory and discover new applications.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Real Analysis and Stochastic Calculus are considered foundational prerequisites, while a basic knowledge of stochastic modelling for finance is helpful yet not necessary.
Examination methods (if applicable)	A 30-minute seminar on one of the selected readings proposed in class
Suggested readings	1. E. Abi Jaber, Lifting the Heston model, Quantitative finance, 19 (2019), pp. 1995–2013. 2. E. Abi Jaber, C. Cuchiero, M. Larsson, and S. Pulido, A weak solution theory for stochastic Volterra equations of convolution type, The Annals of Applied Probability, 31 (2021), pp. 2924–2952. 3. E. Abi Jaber and O. El Euch, Markovian structure of the Volterra Heston model, Statistics & Probability Letters, 149 (2019), pp. 63–72. 4. E. Abi Jaber and O. El Euch, Multifactor approximation of rough volatility models, SIAM journal on financial mathematics, 10 (2019), pp. 309–349. 5. E. Abi Jaber, E. Miller, and H. Pham, Linear-quadratic control for a class of stochastic Volterra equations: solvability and approximation, The Annals of Applied Probability, 31 (2021), pp. 2244–2274.

6. F. E. Benth, N. Detering, and P. Kruehner, Stochastic Volterra integral equations and a class of first-order stochastic partial differential equations, Stochastics, 94 (2022), pp. 1054–1076.
7. M. A. Berger and V. J. Mizel, Volterra equations with Itô integrals -i&ii, The Journal of Integral Equations, (1980), pp. 187–245, 319–337.
8. O. Bonesini, A. Jacquier, and A. Pannier, Rough volatility, path-dependent PDEs and weak rates of convergence. ArXiv preprint, (2024).
9. P. Carmona, L. Coutin, and G. Montseny, Approximation of some gaussian processes, Statistical inference for stochastic processes, 3 (2000), pp. 161–171.
10. L. Coutin and P. Carmona, Fractional Brownian motion and the Markov property, Electronic Communications in Probability, 3 (1998), p. 12.
11. L. Coutin and L. Decreusefond, Stochastic Volterra equations with singular kernels, in Stochastic analysis and mathematical physics, Springer, 2001, pp. 39–50.
12. C. Cuchiero and J. Teichmann, Markovian lifts of positive semidefinite affine Volterra-type processes, Decisions in Economics and Finance, 42 (2019), pp. 407–448.
13. C. Cuchiero and J. Teichmann, Generalized Feller processes and Markovian lifts of stochastic Volterra processes: the affine case, Journal of Evolution Equations, 20 (2020), pp. 1301–1348.
14. J. Gatheral, T. Jaisson, and M. Rosenbaum, Volatility is rough, Quantitative finance, 18 (2018), pp. 933–949.
15. Y. Hamaguchi, Markovian lifting and asymptotic log-Harnack inequality for stochastic Volterra integral equations. ArXiv preprint, (2023).
16. Y. Hamaguchi, Weak well-posedness of stochastic Volterra equations with completely monotone kernels and non-degenerate noise. ArXiv preprint, (2023).
17. P. Harms and D. Stefanovits, Affine representations of fractional processes with applications in mathematical finance, Stochastic Processes and their Applications, 129 (2019), pp. 1185–1228.
18. F. Huber, Markovian lifts of stochastic Volterra equations in Sobolev spaces: Solution theory, an Itô formula and invariant measures, ArXiv preprint, (2024).
19. E. A. Jaber, C. Illand, et al., Joint SPX-VIX calibration with gaussian polynomial volatility models: deep pricing with quantization hints. ArXiv preprint, (2022).
20. L. Mytnik and T. S. Salisbury, Uniqueness for Volterra-type stochastic integral equations, ArXiv preprint, (2015).
21. D. J. Prömel and D. Scheffels, On the existence of weak solutions to stochastic volterra equations, Electronic Communications in Probability, 28 (2023), pp. 1–12.
22. D. J. Prömel and D. Scheffels, Stochastic Volterra equations with Hölder diffusion coefficients, Stochastic Processes and their Applications, 161 (2023), pp. 291–315.
23. D. J. Prömel and D. Scheffels, Pathwise uniqueness for singular stochastic Volterra equations with Hölder coefficients, Stochastics and Partial Differential Equations: Analysis and Computations, (2024), pp. 1–59.
24. F. Viens and J. Zhang, A martingale approach for fractional Brownian motions and related path dependent PDEs, The Annals of Applied Probability, 29 (2019), pp. 3489–3540.

25. H. Wang, J. Yong, and J. Zhang, Path dependent Feynman–Kac formula for forward backward stochastic Volterra integral equations, Annales de l'Institut Henri Poincaré: Probabilités et Statistiques, 58 (2022), pp. 603–638.
26. Z. Wang, Existence and uniqueness of solutions to stochastic Volterra equations with singular kernels and non-Lipschitz coefficients, Statistics & probability letters, 78 (2008), pp. 1062–1071.

Additional information

Course unit English denomination	**Topics in the Calculus of Variations**
SS	MATH-03/A
Teacher in charge (if defined)	Caravenna Laura
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	November 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	The calculus of variations is a cornerstone of mathematical analysis and optimization, focusing on finding functions that minimize or maximize certain quantities, typically expressed as integrals. This theory underpins much of modern physics and engineering, providing the mathematical framework for principles like the least action in mechanics and the optimal shapes and configurations in structural design. It has broad applications, from determining geodesics and surfaces of minimal area to optimizing control systems and studying the behaviour of complex systems described by partial differential equations. In addition to its classical uses, calculus of variations has fueled advances in fields like materials science, image processing, and machine learning, where problems often require identifying optimal configurations or shapes. Optimal transport theory is a relatively new branch of the calculus of variations. The original question involves finding the most efficient ways to move distributions of mass or probability from one configuration to another, while minimizing a given cost function. Initially formulated by Gaspard Monge in 1781 and later generalized by Leonid Kantorovich in 1940-42, it has become a powerful tool both for pure mathematics and for applied problems of redistributeon and matching in economics, physics, data science, and beyond. The first part of the course covers foundational aspects. As a warm-up, I will introduce one-dimensional variational problems and optimality conditions like Euler-Lagrange equations through classical examples, such as the geodesic problem. I will then discuss Monge's formulation of Optimal Transport Problems and its limitations, leading to Kantorovich's relaxation of the problem: here the existence of minimizers directly follows from the direct method of the calculus of

	variations. I will introduce Kantorovich-Rubinstein duality, and necessary and sufficient conditions for optimality. There will be a particular focus on c-cyclical monotonicity, relating it to classical concepts in convex analysis that it generalizes. I will then discuss the problem of existence of optimal maps with a special focus on Brenier's theorem for the quadratic cost function. After this introductory part, I will select applications of optimal transport, also depending on the interest of the audience. Possible choices include: connection with the Monge-Amp`ere equation, Wasserstein distances and their properties, curves in Wasserstein spaces and their relation to the continuity equation, geodesics, Benamou-Brenier formula, characterization of AC curves inWasserstein spaces, introduction to gradient flows in metric spaces and to the JKO minimization scheme for some evolution equations, price equilibria in economic models.
Learning goals	With the first part of the course students will learn the foundational aspects in the Calculus of Variations. The second part will allow them to master more specialized tools from the branch of optimal transport in their applications, also to some evolution PDEs.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Real analysis, some notions of basic PDEs and some functional analysis are welcome, for instance, chapters 1, 3, 4, 8, and 9 of Brezis' book on functional analysis. The main required tools will be recalled during the course.
Examination methods (if applicable)	Oral exam, based either on a problem set or on a research paper
Suggested readings	Notes from lectures will be available. Relevant books for consultation are • A. Figalli, F. Glaudo: An Invitation to Optimal Transport, Wasserstein Distances & Gradient Flows, 2022 • L. Ambrosio, E. Bru´e and D. Semola: Lectures on Optimal Transport, Springer, 2022 • F. Santambrogio: Euclidean, Metric, and Wasserstein Gradient Flows: an overview, Bulletin of Mathematical Sciences, available online, 2017 • F. Santambrogio: Optimal Transport for Applied Mathematicians, Birkhauser, 2015 • L. Ambrosio, N. Gigli: A User's Guide to Optimal Transport, 2012 • L. Ambrosio, N. Gigli, G. Savar´e: Gradient Flows in Metric Spaces and in the Space of Probability Measures, Birkhauser, 2005

C. Villani: Topics in Optimal Transportation, American Mathematical Society, 2003

Additional information

Course unit English denomination	**The Drinfeld double of a finite group**
SS	MATH-02/A – MATH-02/B
Teacher in charge (if defined)	Carnovale Giovanna
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	January 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Basic notions on representations and characters; • Hopf algebras and tensor products of representations. Quasitriangular Hopf algebras and the quantum Yang-Baxter equation; • The different realizations of the Drinfeld double *D(G)* of a group *G*; • Different realizations of the representations of *D(G)*; • The braid group; knot and link invariants from representations of *D(G)*; • Mapping class groups: the case of the torus, and representations of $SL_2(Z)$ obtained from *D(G)*; • Fourier transforms for *G* and *D(G)*; Verlinde formula for the decomposition of the tensor product of representations of *D(G)* • Cibils and Rosso's classification of path algebras with a Hopf algebra algebra structure.
Learning goals	The goal of the course is to offer a glimpse of the Drinfeld double of a finite group and some of its applications in representation theory and topology. It should serve as a tool to see how changing point of view on the same mathematical object can lead to unexpected results. Expected knowledge, abilities and competences We expect that through the rich example of D(G), the participants will acquire familiarity with standard ideas from Hopf algebra theory, such as representations, tensor products, braidings, their potential applications in topology.

Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	The course will review some of the key features of the Drinfeld double of a finite group, its representations, and their applications in topology, developing the theory from scratch and relying for a big part on the basic treatment in [2]. Prerequisites are: basic notions of linear algebra (including the tensor product of vector spaces) and algebra covered in a standard bachelor in mathematics. No prior knowledge of Hopf algebras or representation theory are required. Hopf algebraic tecnichalities will be kept to a minimum.
Examination methods (if applicable)	Solutions of some exercises during the course, followed by an oral discussion
Suggested readings	[1] N. Andruksiewitsch, H.J. Schneider, On the classification of finite-dimensional pointed Hopf algebras, Ann. Math. 171(1), (2010), 375–417. [2] M. Broué, From Rings and Modules to Hopf Algebras. One Flew Over the Algebraist's Nest, Springer Nature Switzerland, (2024). [3] G. Carnovale, N. Ciccoli, E. Collacciani, The versatility of the Drinfeld double of a finite group, survey, Arxiv:2410.11978. [4] C. Cibils, M. Rosso, Algèbres des chemins quantiques, Adv. Math. 125, 171–199 (1997). [5] R. Dijkgraaf, C. Vafa, E. Verlinde, H. Verlinde, The operator algebra of orbifold models, Comm. Math. Phys. 123(3), 485–526, (1989). [6] V.G. Drinfel'd, Quantum groups, in: Proceedings of the I.C.M., Berkeley, (1986), American Math. Soc., 1987, 798?820. [7] T.H. Koornwinder, B. J. Schroers, J. K. Slingerland, F. Bais, Fourier transform and the Verlinde formula for the quantum double of a finite group, Journal of Physics A Mathematical and General 32(48),8539–8549, (1999). [8] G. Lusztig, Characters of Reductive Groups over a Finite Field, Princeton University Press (1984). [9] G. Mason, The quantum double of a finite group and its role in conformal field theory. In: Groups '93 Galway/St. Andrews, 2, 405–417. London Math. Soc. Lecture Note Ser., 212, Cambridge University Press, Cambridge, (1995).
Additional information	**Introduction:** In his 1986 Fields Medal paper [6], Drinfeld introduced the notion of a ring with special features nowadays called the Drinfeld double, one of the main goals being the production of solutions of the quantum Yang-Baxter equation from statistical mechanics. The construction is inspired by a similar construction on Poisson-Lie groups and requires an initial datum given by a general Hopf algebra. The case in which the starting datum is a finite group G is already extremely rich: the double *D(G)* occurs in the work of Dijkgraaf, E. Verlinde, H. Verlinde and Vafa [5]

in conformal field theory, in Lusztig's work on representations of finite groups of Lie type [8], in the study of mapping class groups of surfaces and knot and link invariants, in Verlinde's method to count morphisms from fundamental groups of surfaces to a given group *G*, [9], in Andruskiewitsch and Schneider's program for the classification of Hopf algebras [1], in Cibils and Rosso's classification of path algebras with a Hopf algebra structure, [4]. It is usually studied through its representations, that is, through the different ways in which we can see the elements of *D(G)* as endomorphisms of a given vector space. Representations for *D(G)* can be interpreted in different ways: for example their geometric interpretation in terms of vector bundles lead to the non-abelian Fourier transform in [8]. Verlinde provided a striking formula of its fusion rules (decomposition of tensor products of representations) in terms of group theoretical data making use of conformal field theory only [9]: an algebraic proof of this formula can be given in terms of Fourier transforms on *G x G*, [7]. Several further applications and interpretations of the representations of *D(G)* are listed in the survey [3].

Course unit English denomination	**Stochastic optimal control**
SS	MATH-03/B
Teacher in charge (if defined)	Alekos Cecchin, Markus Fischer
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	February 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Introduction to the classical theory of stochastic control problems with some motivating example from economics and finance. These problems consist in minimizing a cost in which the state variable is given by a controlled stochastic differential equation driven by a Brownian motion. The course will cover the following topics: • Dynamic programming principle: value function, Hamilton-Jacobi-Bellman equation, verification theorem, solutions of second order fully nonlinear PDEs; • Backward stochastic differential equations: representation of the value function for the weak formulation, equivalence of strong and weak formulation, necessary conditions for optimality given by the stochastic Pontryagin's maximum principle, relation with dynamic programming equation; • Linear-Quadratic-Gaussian optimal control problems. Applications to economic and financial models..
Learning goals	Introduce the classical tools to analyze stochastic optimal control problems, such as dynamic programming, Pontryagin maximum principle, Backward SDEs, and then use these methods to study some applications.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic knowledge of stochastic calculus (Brownian motion, stochastic differential equations, filtrations, martingales, ...), as presented, for example, in the course on stochastic analysis of the mater degree. Some concepts will be recalled during the course.
Examination methods (if applicable)	Oral presentation of a research paper related to the topics covered in the course, based on student's interest.
Suggested readings	
Additional information	

Course unit English denomination	**Nonholonomic Mechanical Systems**
SS	MATH-04/A
Teacher in charge (if defined)	Francesco Fassò
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	January 2026 (TBC)
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Nonholonomic constraints. Distributions and Frobenius theorem. Ideal constraints and D'Alembert principle. The equations of motion for linear and nonlinear constraints. The role and structure of the reaction forces. Basic examples of nonholonomic mechanical systems. First integrals: energy, momenta, the nonholonomic Noether theorem. Based on the interest of the partecipants and on the time availability, one or more of the following topics might be covered: (non)existence of conserved measures; reduction under symmetry; integrability; control and trajectory planning of nonholonomic systems.
Learning goals	The aim of the course is to provide a basic, though solid, knowledge of the geometric structure and of the dynamics of nonholonomic mechanical systems, leading the students to the level of understanding the current literature.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No

Prerequisites (not mandatory)	Prerequisites are basic notions of classical mechanics (Lagrangian mechanics), of the qualitative theory of ODEs and of analysis and differential geometry. More advanced topics (e.g.,Frobenius theorem, Lie groups actions) will be reviewed.
Examination methods (if applicable)	Verification of the comprehension and knowledge of the core topics covered in the course through a colloquium and/or the presentation of specific topics studied individually.
Suggested readings	1. Lecture notes will be prepared and made accessible to the students during the course. 2. L. Garcìa-Naranjo, *Geometry and dynamics of nonholonomic systems.* Lecture Notes for a course given in this Doctoral School in 2021. 3. Ju. I. Neĭmark and N.A. Fufaev, *Dynamics of Nonholonomic Systems* (AMS, 1972). 4. F. Bullo and A.D. Lewis, *Geometric Control of Mechanical Systems* (Springer, 2004). 5. R. Cushman, J.J. Duistermaat and J. Snyaticki, *Geometry of Nonholonomically Constrained Systems* (World Scientific, 2010). 6. A. Bloch, *Nonholonomic Mechanics and Control* (Springer, 2015).
Additional information	**Introduction:** In mechanics, a nonholonomic constraint is a restriction on the possible velocities of a holonomic system without restricting its possible configurations. Examples are systems of rigid bodies whose contact points are constrained to have a preassigned velocity and the possibility to manoeuvre a car into parallel parking despite the fact that the wheels cannot move transversally. Nonholonomic mechanics is very different from Lagrangian mechanis because the equations of motion do not come from a variational principle. Many properties of the dynamics are still not completely understood and the subject is an active field of research, which is of interest in various areas of Mathematics and Engeneering. The aim of the course is to give an introduction to the subject, focussing mostly on the dynamical aspects and presenting the students to recent progress and open questions in the field.

Course unit English denomination	**Geometric aspects of PDEs**
SS	MATH-03/A
Teacher in charge (if defined)	Fogagnolo Mattia, Franceschi Valentina
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	March 2026 (TBC)
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	In this course we deal with some fundamental properties of basic elliptic PDEs, and build on them to discuss powerful results in the geometric analysis and regularity theory of submanifolds. The techniques and the presentation are suited to be applied in more general geometries and to more general equations. After recalling some basics on the geometry of submanifolds and the regularity theory for elliptic pdes, the topics treated will include: • Sharp gradient bounds for solutions to Laplace equations, leading in turn to the characterization of spheres as the only closed surfaces with constant mean curvature (Alexandrov theorem). This result and its proof will then be compared with Serrin's overdetermined problem. • Regularity of solutions to elliptic PDEs, minimal graphs and regularity of sets with bounded mean curvature. Monotonicity formula for minimal surfaces and Allard's Theorem. • Almgren's frequency function and estimates of the critical set of harmonic functions, in connection with the monotonicity formula for stationary submanifolds.
Learning goals	The course aims at providing the attendees some deep connections among fundamental properties of elliptic PDEs and prototypical problems in geometric analysis.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basics in PDEs, calculus of variations, geometry of submanifolds.
Examination methods (in applicable)	Seminar about a research paper in the topic.
Suggested readings	W. K. ALLARD On the first vatiation of a varifold, Ann. of Math., 1972. E. BOMBIERI, E. DE GIORGI, M. MIRANDA Una maggiorazione a priori relativa alle ipersuperfici minimali non parametriche, ARMA. 1969. X. FERNANDEZ-REAL, X. ROS-OTON Regularity Theory for Elliptic PDEs, EMS Press, 2022. D. GILBARG, N. TRUDINGER Elliptic Partial Differential Equations of Second Order, Springer, 1997. A. NABER, D. VALTORTA Volume Estimates on the Critical Sets of Solutions to Elliptic PDEs, CPAM. 2017. R. C. REILLY Mean curvature, the Laplacian, and soap bubbles, American Mathematical Monthly, 1982. Y. TONEGAWA Brakke's Mean Curvature Flow An Introduction Springer, 2019. X.-J. WANG Interior gradient estimates for mean curvature equations, Math. Z., 1998. B. WHITE A local regularity theorem for mean curvature flow, Ann. of Math., 2005.
Additional information	

Course unit English denomination	**Lie Groups and Symmetry**
SS	MATH-04/A
Teacher in charge (if defined)	Luis C. García-Naranjo
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	November 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Lie groups and their differential and group structures (left and right trivializations, Lie algebra of a Lie group, homomorphisms, exponential map, (co)adjoint action), structure of compact Lie groups (maximal tori, Weyl chambers); classical matrix groups and their properties; relationship between Lie groups and Lie algebras, Lie's Theorems and the Baker-Campbell-Hausdorff formula; differentiable actions of Lie groups on manifolds, quotient spaces (for proper actions), Palais slice theorem, invariant vector fields; reduction of invariant vector fields; applications to ODEs with symmetry (relative equilbria and relative periodic orbits, connection with Floquet theory, reduction and reconstruction, integrability).
Learning goals	The course aims at providing an introduction to the theory of Lie groups and their actions, which is a topic of broad interest in several areas of Mathematics and applications. After covering the fundamentals of the subject, the course will provide some examples of use of Lie groups in the study of ODEs with symmetry.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No

Prerequisites (not mandatory)	Basic knowledge of differential geometry. The course is addressed to all students.
Examination methods (if applicable)	Oral examination on the topics covered during the course.
Suggested readings	1. J.J. Duistermaat, J.A.C. Kolk, Lie Groups. (Springer, 2000). 2. Baker, Matrix groups. An introduction to Lie group theory. (Springer, 2002) 3. J. Lee, Introduction to Smooth manifolds. 2nd edition. (Springer, 2013) 4. R. Cushman, J.J. Duistermaat and J. ´Snyaticki, Geometry of Nonholonomically Constrained Systems. (World Scientific, 2010).
Additional information	

UNIVERSITÀ DEGLI STUDI DI PADOVA

SCIENZE MATEMATICHE

Course unit English denomination	**Graphs, algebras and representations**
SS	MATH-02/A
Teacher in charge (if defined)	Daniel Labardini-Fragoso, Jorge Vitória
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	November 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	● Quivers, path algebras, quotients of path algebras and their representations. ● Modules vs. Representations. ● Basic properties of algebras coming from graphs and their representations. ● Some classes of algebras: finite-dimensional algebras; Leavitt path algebras; Jacobian algebras of quivers with potentials; incidence algebras of posets. ● Elements of the representation theory of one of the preceding classes: – projective, injective and simple representations; – combinatorial and homological properties; – categorical equivalences; – classification results.
Learning goals	At the end of the course, students will be able to: • Identify basic structural properties of algebras coming from graphs and their representations; • Understand some links between the combinatorics of a quiver and the representation theory of the corresponding path algebra (and its quotients); • Learn standard (combinatorial, homological, categorical or geometric) techniques in the representation theory of some classes of algebras;
Teaching methods	Frontal lectures

Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Students following this course should have followed courses in linear algebra and undergraduate abstract algebra (namely an introduction to rings). A basic knowledge of categories and functors is useful but not required.
Examination methods (if applicable)	The exam will consist on a seminar covering a part of a research paper related to the course.
Suggested readings	• Abrams, G., Ara, P. and Molina, M.S., Leavitt Path Algebras, Lecture Notes in Mathematics 2191, Springer (2017). • Assem, I., Skowronski, A. and Simson, D., Elements of the Representation Theory of Associative Algebras: Techniques of Representation Theory, Cambridge University Press (2006). • Derksen, H., Weyman, J. and Zelevinsky, A., Quivers with potentials and their representations I: Mutations, Selecta Math. 14 (2008), no. 1, 59–119. • Rota, G., On the foundations of combinatorial theory. I. Theory of M¨obius functions, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete (1964), 340–368.
Additional information	

UNIVERSITÀ DEGLI STUDI DI PADOVA

SCIENZE MATEMATICHE

Course unit English denomination	**Intersection Theory**
SS	MATH-02/B
Teacher in charge (if defined)	Jakob Scholbach
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	February 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Chow groups, including higher Chow groups • K-theory • Characteristic classes • The theorem of Grothendieck-Riemann-Roch ● Introduction to motivic sheaves, six functor formalisms
Learning goals	The aim of the course is to introduce the students to intersection theory and to give a first idea of the theory of motivic sheaves.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Algebraic geometry
Examination methods (if applicable)	There will be an oral exam at the end of the course.

Suggested readings	1. Eisenbud and Harris: “3264 and all that—a second course in algebraic geometry.” 2. Fulton: “Intersection theory” 3. Cisinski and Déglise: “Triangulated categories of motives”
Additional information	

Course unit English denomination	**Generic Structures in PDEs, Control, and Games**
SS	MATH-03/A
Teacher in charge (if defined)	Khai T. Nguyen
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	May 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Controlled scalar balance laws: Transversality theorem, generic regularity properties and quantitative estimate on total number of shocks • The Bolza problem: Existence of optimal solutions, Pontryagin's Maximum Principle, and a priori estimates • Lipschitz continuity of generalized monotone operators and the ε-entropy of solution sets • Necessary conditions for conjugate points and generic uniqueness for optimal solutions • Sharp quantitative estimate for critical sets and zeroes of multivariable polynomials • Generic properties of first order mean field games
Learning goals	This course is to explore foundational and advanced topics in nonlinear partial differential equations, optimal control, and mean field games. We will focus on generic regularity, transversality, shock formation, and quantitative properties of solution sets in both analytical and applied contexts.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Analysis, linear algebra and measure theory
Examination methods (if applicable)	Oral presentation of a research paper
Suggested readings	• A. Bressan and B. Piccoli, Introduction to the Mathematical Theory of Control, AIMS Series in Applied Mathematics, Springfield Mo. 2007. • A. Bressan and K. T. Nguyen, Generic properties of mean field games, Dynamic Games Appl. 13 (2023), 750–782. • A. Bressan, M. Mazzola, and K.T. Nguyen, Generic uniqueness and conjugate points for optimal control problems, Arxiv: https://arxiv.org/abs/2501.10572 • A. Bressan and K.T. Nguyen, Generic solutions to controlled balance laws Arxiv:https://arxiv.org/abs/2410.20032 • A. Murdza and K. T. Nguyen, A quantitative version of the transversality theorem, Communications in Mathematical Sciences 21 (2023), no. 5 1302-1320 • A. Murdza and K. T. Nguyen, A sharp quantitative estimate of critical sets Arxiv: https://arxiv.org/abs/2405.17107 • M. Golubitsky and V. Guillemin, Stable Mappings and their Singularities. SpringerVerlag, New York, 1973.
Additional information	

Course unit English denomination	**Advanced Monte Carlo methods with applications to filtering theory**
SS	MATH-03/B, STAT-04/A
Teacher in charge (if defined)	Pierre Del Moral
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	May 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• **PART 1** [4h] - Introduction: the optimal filtering problem; linear and non-linear filtering problems; the Bayesian framework; Kalman filter and extensions; numerical methods for filtering. • **PART 2** [6h] - Linear Monte Carlo methods: Markov chain Monte Carlo methods; numerics for signal processing: Kalman filters, backward smoothing, hidden Markov chains. • **PART 3** [6h] - Particle filtering and sequential Monte Carlo methods: introduction, computational efficiency, implementation challenges and recent applications.
Learning goals	This course will cover topics in the general area of Monte Carlo methods and their application domains, with a special emphasis on numerical methods for stochastic filtering and signal processing. The topics include Markov chain Monte Carlo (MCMC) and Sequential Monte Carlo methods (SMC), as well as branching and interacting particle methodologies. The lectures cover discrete and continuous time stochastic models, starting from traditional sampling techniques (perfect simulation, Metropolis-Hasting, and Gibbs-Glauber models) to more refined methodologies such as gradient flows diffusions on constraint state space and Riemannian manifolds, ending with the more recent and rapidly developing Branching and mean field type Interacting Particle Systems techniques. The final part of the lectures will focus on particle methods for filtering and covers forward/backward particle filters, extended and Ensemble Kalman filers and unscented Kalman filters. The course offers a pedagogical introduction to the theoretical foundations of these advanced stochastic models, combined with a series of concrete

	illustrations taken from different application domains. The applications considered in these lectures will range from Bayesian statistical learning (hidden Markov chain, statistical machine learning), risk analysis and rare event sampling (mathematical finance, and industrial risk assessment), operation research (global optimization, combinatorial counting and ranking), avanced signal processing (stochastic nonlinear filtering and control, and data association and multiple objects tracking), computational and statistical physics (Feynman-Kac formulae on path spaces, molecular dynamics, Schrödinger's ground states, Boltzmann-Gibbs distributions, and free energy computation). Approximately the first half of the course will be concerned with linear type Markov chain Monte Carlo methods, and the second part to nonlinear particle type methodologies, including interacting diffusions, interacting jump processes and genealogical tree based samplers. A list of topics intended to be covered is attached.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Probability and stochastic calculus
Examination methods (if applicable)	Seminar on a relevant pape
Suggested readings	Self-contained and detailed lecture notes for the course will be provided. Other textbooks which can be useful for supplemental reading are: **References:** • Stochastic Processes: From Applications to Theory. P. Del Moral, & S. Penev Chapman and Hall/CRC (2017). • Mean field simulation for Monte Carlo integration. P. Del Moral. Chapman & Hall/CRC Monographs on Statistics & Applied Probability (2013). • Feynman-Kac formulae. Genealogical and interacting particle approximations. P. Del Moral. Springer New York. Series: Probability and Applications (2004). • Fundamentals of Stochastic Filtering. A. Bain and D. Crisan. Springer, Stochastic Modelling and Applied Probability, Vol. 60 (2009). ● • Inference in Hidden Markov Models. O. Capp´e, E. Moulines, and T. Ryden. Springer series in Statistics (2005).
Additional information	

Course unit English denomination	**An introduction to free boundary problems**
SS	MATH-03/A
Teacher in charge (if defined)	Guido De Philippis
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	January 2026
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	The course will introduce the basic existence and regualirty theory for solutions to the obstacle and Bernoulli problem. If time allows the structure of singularities will also be investigated.
Learning goals	Goal of the course will be to present basic techniques in the study of free boundary problems, this will be done by studying “prototypical” problems like the Obstacle Problem and the Bernoulli Problem.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Some exposition to basic PDE (mostly basic properties of harmonic functions) and to Sobolev spaces (Sobolev/Poincarè inequalities, trace theorems,...) is advised.
Examination methods (if applicable)	Examination will be based on students presentation

Suggested readings	• L. A. Caffarelli}, The obstacle problem. Rome: Accademia Nazionale dei Lincei; Pisa: Scuola Normale Superiore (1998; Zbl 1084.49001) • L. A. Caffarelli}, J. Fourier Anal. Appl. 4, No. 4--5, 383--402 (1998; Zbl 0928.49030) • B. Velichkov}, Regularity of the one-phase free boundaries. Cham: Springer; Bologna: Unione Matematica Italiana (UMI) (2023; Zbl 1558.35007)
Additional information	