Course unit English denomination	**The minimal action principle and applications**
Teacher in charge (if defined)	Luca Baracco, Olga Bernardi
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	Second semester
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	For a Lagrangian on a tangent bundle, the Euler-Lagrange equations encode the variational character of Lagrangian Mechanics. In fact, it is well-known that a curve solves E-L if and only if the first variation of the corresponding action with fixed extremals, vanishes. Concerning minimality, in general it holds for short times. In fact, due to the possible presence of conjugated points, critical curves cease to be minimizing for larger times. Only under certain convexity assumptions there are "Action minimizing orbits". For such a distinguished and mechanical relevant class of Lagrangian -the so-called Tonelli Lagrangians- the Legendre transform is a global diffeomorphism and E-L equations equal to Hamilton's equations for the corresponding Hamiltonian. For autonomous systems, gives the conserved energy value along a solution. Beyond Lagrangian and Hamiltonian setting, the search of dynamically relevant minimal objects is one of the central topics of modern theory of Dynamical Systems. One of the first results in this direction go back to the Eighties with the so-called Aubry-Mather theory for monotone twist map. An important application of this theory is the study of mathematical billiards, from Birkhoff to -more recent- types of billiards like symplectic and outer billiards. The generalization of such a theory from one to more degrees of freedom have been developed two decades later with Mather-Mané theory, where minimizing measures, instead of trajectories, play a crucial role. This significant theory has connections from Hamilton-Jacobi equation to Symplectic Topology.The aim of this PhD course is to present -in a self-contained way- the "Minimal Action Principle" in different settings. This principle can be considered a sort of -largely accepted"thriftiness" of Nature in its motions.
Learning goals	A sample of various settings, also beyond classical Lagrangian and Hamiltonian framework, where the “Least Action Principle” play a fundamental role. The course includes basic notions and proofs of Aubry-Mather Theory and Mather-Man´e Theory. Moreover, we discuss connections both with mathematical billiards and Symplectic Topology.

Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic notions of Mathematical Physics and Calculus of Variations
Examination methods (in applicable)	Oral exam
Suggested readings	• Siburg K.F. The principle of least action in geometry and dynamics. Lecture Notes in Mathematics, vol. 1844, xiii+ 128 pp. Berlin, Germany: Springer, (2004). • Bangert V. Mather Sets for Twist Maps and Geodesics on Tori Dynamics Reported, Volume 1 Dynamics Reported, (1988). • Paternain G.P. Polterovich L. Siburg K.F. Boundary rigidity for Lagrangian submanifolds, non-removable intersections, and Aubry-Mather theory, Volume 3, Number 2, (2003). • Sorrentino A. Action-minimizing methods in Hamiltonian dynamics: an introduction to AubryMather theory. Monograph in the Series: Mathematical Lecture Notes Vol. 50, Princeton University Press, (2015). • Tabachnikov S. Geometry and Billiards (Student Mathematical Library) (Student Mathematical Library, 30), (2005).
Additional information	

Course unit English denomination	**Curves in Hilbert Modular Surfaces**
Teacher in charge (if defined)	Gabriele Bogo
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	1. The moduli space A_g of abelian varieties of dimension g and the locus of real multiplication. Hilbert modular varieties (an overview). 2. Kobayashi metric on A_g and totally geodesic curves (Kobayashi curves). They are defined over number fields and lie in the locus of real multiplication. 3. Further motivation: Belyi's theorem and Ellenberg-McReynolds's result ("every curve is a Teichmüller curve"). 4. Kobayashi curves as curves in Hilbert modular varieties and modular embeddings. Arithmetic Fuchsian groups. 5. Example I. Totally geodesic curves in A_2 for the Riemannian metric: Hirzebruch-Zagier curves. Their relation with modular curves and moduli interpretation. 6. Teichmüller space and Teichm¨uller metric; mapping class group and moduli space of curves (an overview). 7. Example II. Totally geodesic curves in A_2 for the Teichmüller metric: Teichmüller curves. Their relation with rational billiards (flat surfaces) and moduli interpretation. 8. If time permits, special (complex multiplication) points in the arithmetic and nonarithmetic cases.
Learning goals	Belyi theorem states that every algebraic curve defined over Q is a branched cover of the projective line minus three points. Such a curve is, by Poincaré's unifomization theorem, a (possibly compactified) quotient H/Γ of the upper half-plane by a discrete group Γ _ $SL_2(R)$. In some cases, the group Γ is *arithmetic*, i.e., it comes from a algebra (the curve is then a modular or Shimura curve), but in most cases it is not. An important example of non-arithmetic curves is provided by Teichmüller curves, a class of curves first discovered in relation with the dynamics of rational billiard tables. The first aim of this course is to provide a unified treatment of algebraic curves over Q as totally geodesic curves in the moduli space of abelian surfaces, with respect to the Kobayashi metric. The second aim is to discuss in detail two classes of such curves, the Hirzebruch-Zagier curves, and the

	Teichmüller curves in genus two. If time permits, we will discuss the special points on these curves.
Teaching methods	Face-to-face/Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic algebraic number theory and algebraic geometry (algebraic varieties). Previous exposure to the concept of moduli spaces is helpful but not required.
Examination methods (if applicable)	Seminar about a research paper on the subject
Suggested readings	1. Hirzebruch, F., Zagier, D. Intersection numbers of curves on Hilbert modular surfaces and modular forms of Nebentypus, Invent. Math. 36(1976), 57–113. 2. McMullen, C. Billiards and Teichmüller curves on Hilbert modular surfaces, J. Amer.Math. Soc.16(2003), no.4, 857–885. 3. Möller, M., Viehweg, E. Kobayashi geodesics in A_g , J. Differential Geom. 86(2010), no.2, 355–379.
Additional information	

Course unit English denomination	**Introduction to scalar conservation laws**
Teacher in charge (if defined)	Paolo Bonicatto, Elio Marconi Dipartimento di Matematica, Università di Trento Email: paolo.bonicatto@unitn.it Dipartimento di Matematica ”Tullio Levi-Civita”, Università di Padova Email: emarconi@math.unipd.i
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	February – March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	1. Well-posedness of scalar conservation laws in the class of bounded entropy solutions: approximation schemes, Kružkov a priori estimate 2. Kinetic formulation of scalar conservation laws 3. Lagrangian description of entropy solutions
Learning goals	Introduce the theory of entropy solutions of scalar conservation laws. Both the classical theory initiated by Kružkov and the kinetic formulation will be presented.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites *(not mandatory)*	Calculus, measure theory
Examination methods *(if applicable)*	Oral presentation of a research paper
Suggested readings	• Kružkov, S. N., First order quasilinear equations with several independent variables, Matematicheskiĭ Sbornik, 81 (123), 1970. • Perthame, B., Kinetic Formulation of Conservation Laws, Oxford Lecture Series in Mathematics and Its Applications

Additional information

Course unit English denomination	**Interpolation theory for differential forms**
Teacher in charge (if defined)	Ludovico Bruni Bruno Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova and Istituto Nazionale di Alta Matematica Email:ludovico.brunibruno@unipd.it, brunibruno@altamatematica.it
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	November 2024
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• **Introduction and preliminaries:** 1. quick review of discrete differential geometry (from differential forms to polynomial differential forms); 2. quick review of simplices and triangulations (simplices, subsimplices, complexes); 3. quick review of interpolation theory (projectors, degrees of freedom). • **One-dimensional framework:** 1. interpolation by point evaluations on the line; 2. interpolation by integral segments on the line; 3. Lebesgue constant vs generalised Lebesgue constant; 4. Fekete problem. • **Multi-dimensional framework:** 1. Whitney forms; 2. simplicial elements and small simplices; 3. weights: uniform vs non uniform small simplices; 4. the generalised Lebesgue constant for k-forms; 5. application to other shapes (n-balls). • **Applications:** 1. computational aspects; 2. the problem of conditioning in FEM; 3. preconditioning by weights
Learning goals	The principal aim of this course is to understand the main challenges of interpolation of differential forms. To do so, techniques based on integration on simplices (i.e. on the construction of weights) are studied. This involves the identification of unisolvent sets and their comparison in terms of appropriate functionals, the k-Lebesgue constants, that are discussed and characterised. At the end of the course, the student is in a

	position to deal with the existing literature and aware of modern challenges in the field.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Numerical Analysis I, suitable for students with background in analysis or differential geometry and/or approximation theory.
Examination methods (in applicable)	Seminar
Suggested readings	1. L. Bruni Bruno, Weights for high order Whitney finite elements, PhD thesis (2022) 2. F. Rapetti and A. Bossavit, Whitney forms of higher degree, SIAM J. Numer. Anal., 47 (2009) 3. F. Rapetti, High order edge elements on simplicial meshes, ESAIM: M2AN, 41 (2007) 4. R. A. Nicolaides, On a class of finite elements generated by Lagrange interpolation, SIAM J. Numer. Anal., 9 (1972) 5. A. Alonso Rodrìguez and F. Rapetti, On a generalization of the Lebesgues constant, Journal of Computational Physics, 428 (2021) D. N. Arnold, R. S. Falk, and R. Winther, Finite element exterior calculus, homological techniques, and applications, Acta Numer., 15 (2006)
Additional information	

UNIVERSITÀ DEGLI STUDI DI PADOVA

SCIENZE MATEMATICHE

Course unit English denomination	**Stability of Queueing Networks**
Teacher in charge (if defined)	Bernardo D'Auria Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova Email: bernardo.dauria@unipd.it
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	April 28th – May 21st (Mondays and Wednesdays)
Course delivery method	☒ In presence ☐ On line ☐ Dual
Language of instruction	English
Mandatory attendance	☒ Yes (75% minimum attendance) ☐ No
Course unit contents	• Introduction to queueing networks • The classical networks • Instability of Subcritical Queueing Networks • Stability of Queueing Networks
Learning goals	Queueing networks pervade modern systems, playing a crucial role in diverse fields such as telecommunications, computer networks, manufacturing processes, and service systems. This course aims to foster a profound comprehension of these stochastic models, with a specific emphasis on exploring issues related to stability. Following a formal introduction to queueing networks, the course will delve into the intricacies of stability properties. It will shed light on this concept by presenting examples of unstable networks while also introducing techniques for establishing the stability of stochastic networks through the application of fluid models and Lyapunov functions.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites *(not mandatory)*	Basic knowledge of continuous-time Markov Chains
Examination methods *(if applicable)*	Solving the exercises assigned during the course

Suggested readings	• M. Bramson (2008). Stability of queueing networks. Probab. Surv., 5:169-345. DOI:10.1214/08-PS137 • M. Bramson, B. D'Auria and N. Walton (2021). Stability and Instability of the MaxWeight Policy. Math. Oper. Res., 46(4): 1611-1638. DOI:10.1287/moor.2020.1106
Additional information	

Course unit English denomination	**Convex polyhedra and their diameter**
Teacher in charge (if defined)	Marco Di Summa
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	November 2024
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Polyhedra: basic facts, inequality description, vertex/ray description, fundamental examples • The role of polyhedra in several disciplines • The skeleton of a polyhedron and the notion of diameter • Lower and upper bounds on the diameter of a polyhedron • Hirsch conjecture and its link with the efficiency of linear programming algorithms • Correctness of the conjecture for 0/1 polytopes and for some other relevant cases • Santos' counterexample to the Hirsch conjecture (sketch) • The (weaker) polynomial Hirsch conjecture and some related results • (Time permitting) The recent notion of circuit diameter
Learning goals	A convex polyhedron is defined as the set of solutions to a given system of linear inequalities, and as such, it generalizes to n dimensions the commonly known notion of 3-dimensional polyhedron, which is used, for instance, to describe the structure of molecules and chemical compounds. Besides being natural and interesting mathematical objects, general n-dimensional polyhedra are fundamental in the field of optimization. In this course we will introduce some basic properties of polyhedra and discuss their importance and usefulness. After reviewing some fundamental structural results, we will focus on the notion of diameter of a polyhedron. In order to understand what the diameter of a polyhedron is, one can visualize the 3-dimensional case, where the notions of vertex and edge should be clear: The diameter of a polyhedron is the maximum distance between any two vertices, where, for a fixed pair of vertices, the distance is measured by counting the minimum number of steps needed to move from one vertex to the other by traversing edges. (This can be properly extended to the n-dimensional case.) We will survey classical and famous results on the diameter of polyhedra, as well as some recent achievements. We will spend some time on the Hirsch conjecture, which was posed in 1957, proved for some special but very important cases in

	the subsequent decades, and finally disproved by Francisco Santos in 2010 in a paper that won the Fulkerson prize, one of the most important awards in the area of Discrete Mathematics. However, a weaker version, called the polynomial Hirsch conjecture, is still open, and its correctness would be extremely important to assess the efficiency of some algorithms for the solution of linear optimization problems. The topic of this course can be interesting for students in various fields of Mathematics, as it connects (at least) combinatorial and discrete geometry, graph theory, and optimization. Graphs and their diameter are also studied in algebra and probability (e.g., random walks are sometimes the tool to prove the existence of a short path between two vertices).
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☒ Yes ☐ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Standard Mathematical knowledge will be sufficient
Examination methods (in applicable)	Seminar
Suggested readings	For the introductory part on polyhedra, students can refer to Chapter 3 of the following book, also accessible online: • M. Conforti, G. Cornuéjols, G. Zambelli, Integer Programming, Springer, 2014. The notion of diameter and some classical related results are discussed, e.g., in Chapter 3 of the following book: • G. M. Ziegler, Lecture on Polytopes, Springer, 2007. More specific references (including those needed to be prepared for the exam, which is in the form of student seminar) will be provided during the course
Additional information	

Course unit English denomination	**The isoperimetric problem: techniques and applications**
Teacher in charge (if defined)	Mattia Fogagnolo, Valentina Franceschi Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova Email: mattia.fogagnolo@unipd.it Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova Email: valentina.franceschi@unipd.it
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	3,4,6,10,11,13 march 2025 (mon and tue 3 hours, thu 2 hours).
Course delivery method	☐ In presence ☐ Remotely ☒ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	In this course we deal with the isoperimetric problem and the isoperimetric inequality in the Euclidean space. We present a number of related techniques and results, such as the direct method of the calculus of variations, the Schwarz symmetrisation, the ABP method. We will describe the close relation between isoperimetric, Sobolev and Faber-Krahn inequalities. We are going to discuss these and other tools in connection with the isoperimetric problem beyond the Euclidean space, with a focus on Riemannian manifolds with curvature bounds and the Heisenberg group. Time permitting, we will describe mean curvature inequalities, the quantitative isoperimetric inequality, and the minimal partition problems.
Learning goals	The course aims at providing the attendees a number of modern techniques that have been utilized in connection with the isoperimetric problem, with an eye to the analysis of metric spaces and to the open problem in contemporary research.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites	Basics in PDEs and calculus of variations.

(not mandatory)	
Examination methods (in applicable)	Seminar about a research paper in the topic.
Suggested readings	• L. Ambrosio, E. Brué, D. Semola, Lectures on Optimal Transport, Springer, 2021. • G. Antonelli, E. Pasqualetto, M. Pozzetta, D. Semola, Asymptotic isoperimetry on noncollapsed spaces with lower Ricci bounds • X. Cabré , Elliptic PDE's in probability and geometry: Symmetry and regularity of solutions, Discrete and Continuous Dynamical Systems, 2008 • Chavel, Eigenvalues in Riemannian geometry, Elsevier, 1984. • S. Gallot, D. H, J. Lafontaine, Riemannian Geometry, Springer, 2002. • M. Fogagnolo, L. Mazzieri, Minimising hulls, p-capacity and isoperimetric inequality J. Functional Anal. 2022. • V. Franceschi, R. Monti, Isoperimetric Problem in H-type groups and Grushin spaces, Rev. Mat. Iberoam., 2014. • Bruce Kleiner, An isoperimetric comparison theorem, Inventiones Math., 1992 • F. Maggi, Sets of Finite Perimeter and Geometric Variational Problems, Cambridge University Press, 2012. • R. Monti, Heisenberg isoperimetric problem. The axial case, Adv. Calc. Var., 2008 • S. Nardulli, Generalized existence of isoperimetric regions in non-compact Riemannian manifolds and applications to the isoperimetric profile, Asian J. of Math. 2014. • R. Magnanini, G. Poggesi, Serrin's problem and Alexandrov's Soap Bubble Theorem: enhanced stability via integral identities, Indiana Univ. Math. Journal, 2020. • M. Ritoré, Isoperimetric Inequalities in Riemannian Manifolds, Birkh¨auser, 2023. • M. Ritoré, C. Rosales, Existence and characterization of regions minimizing perimeter under a volume constraint inside Euclidean cones, TAMS, 2004. • M. Ritoré, C. Rosales, Area-stationary surfaces in the Heisenberg group H1,Adv. in Math., 2008.
Additional information	

UNIVERSITÀ DEGLI STUDI DI PADOVA

SCIENZE MATEMATICHE

Course unit English denomination	**The Maximal Subgroups of the Symmetric Group**
Teacher in charge (if defined)	Martino Garonzi
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	February/March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Course contents: • Finite permutation groups, • Wreath products, • Transitive and primitive actions, • Minimal normal subgroups and socle of a finite group, • Description of the maximal subgroups of the symmetric groups, O'Nan-Scott theorem for finite primitive permutation groups: affine groups, almost-simple groups, product actions and diagonal actions. • Applications and discussion of relevant recent results.
Learning goals	It is expected that the students achieve the knowledge of the O'Nan-Scott theorem and its proof, so as to make them able to prove results about generation and the subgroup structure of the symmetric group and of permutation groups in general. For example, it is expected that the students understand that the primitive maximal subgroups of S_n (and A_n) represent a "local obstruction": as a corollary of the O'Nan-Scott theorem, the set of numbers n such that S_n (and A_n) presents no such local obstruction has density 1.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	This is a course on finite permutation groups, focusing on the classification of the maximal subgroups of the finite symmetric groups S_n and of the finite alternating groups A_n. The background required is a basic knowledge of group

	theory (including notions such as solvability and nilpotency of groups) and group actions.
Examination methods (if applicable)	Seminar
Suggested readings	1. Cameron, Peter J.; Permutation groups. London Mathematical Society Student Texts, 45. Cambridge University Press, Cambridge, 1999. 2. Dixon, John D.; Mortimer, Brian; Permutation groups. Graduate Texts in Mathematics, 163. Springer-Verlag, New York, 1996. 3. Liebeck, Martin W.; Praeger, Cheryl E.; Saxl, Jan; On the O'Nan-Scott theorem for finite primitive permutation groups. J. Austral. Math. Soc. Ser. A 44 (1988), no. 3, 389–396.
Additional information	

Course unit English denomination	**Products of random matrices: theory and applications**
Teacher in charge (if defined)	Giambattista Giacomin Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova Email: giacomin@math.unipd.it
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	November-December 2024
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	Italian, except if English is requested
Mandatory attendance	☒ Yes (75% minimum of presence) ☐ No
Course unit contents	Products of random matrices appear as a fundamental model and/or tool in a number of mathematical contexts, ranging from very abstract to very applied ones. The theory is rich already in the case in which one deals with products of independent and identically distributed (IID) random d x d matrices $(M_n)_n$=1,2,…, with d an integer larger than 1: one of the main issues in this context is understanding the leading asymptotic behavior, for $n \to \infty$, of log $\|M_1 M_2 \ldots M_n\|$, with $\|\cdot\|$ a matrix norm, assuming that log $\|M_1\|$ is in L^1. This is the natural generalization of the case in which d = 1, where $M_1 M_2 \ldots M_n$ is just a product of IID real random variable and the issue we raise is easily solved by applying the Law of Large Numbers, and the answer is very explicit. The situation is more involved when d = 2 or more: in fact, the problem we just raised is the identification of the top Lyapunov exponent of the product of random matrices we consider and, while the theory is very complete, the answer is really not trivial and definitely not as explicit as for d = 1. It involves in particular the construction of an auxiliary process that is interesting in its own right. In the first part of the course we develop the theory that leads to a formula, the Furstenberg formula, for the top Lyapunov exponent. While some results will be given (and proven) for general d, the full theory will be developed only for d = 2. We will also study the second Lyapunov exponent (i.e., for d = 2 we will study all the Lyapunov exponents, since there are d Lyapunov exponents): the key result that we will prove is Oseledets Theorem. The second part of the course focuses on applications. Possible topics include: • Anderson localization in one dimension: the Schrödinger operator with random potentials

	• The transfer matrix method in statistical mechanics and application to some disordered models. In reality the two parts of the course will be to a certain extent entangled: the random matrices that are relevant for the applications will be used as examples in the theoretical part of the course.
Learning goals	Learning fundamentals of products of random matrices
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☒ Yes ☐ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	A first course in probability with measure theory. Some knowledge of stochastic processes with discrete time (Markov chains, martingales) is welcome, but the important concepts will be (re)introduced in some detail.
Examination methods (in applicable)	Oral exam
Suggested readings	There will be lecture notes, mostly based (for the first part of the course) on • P. Bougerol and J. Lacroix, Products of random matrices with applications to Schrödinger operators, Progress in Probability and Statistics, 8, Birkhäuser, 1985. • M. Viana, Lectures on Lyapunov exponents, Cambridge Studies in Advanced Mathematics, 145 Cambridge University Press, 2014.
Additional information	

Course unit English denomination	**Special Functions and Applications**
Teacher in charge (if defined)	Giulio G. Giusteri Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova Email: giulio.giusteri@unipd.it
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	January 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Recap of basic facts in complex analysis: holomorphic functions, Laurent series, contour integrals, Cauchy theorem. Euler's Gamma function. • The Probability Integral: from error estimates to heat conduction and boundary layers. • The Heat equation and the Laplace transform. • Legendre and Hermite polynomials: Schrödinger equation and the quantum harmonic oscillator. • Curvilinear coordinates. Laplace equation, separation of variables. Laguerre polynomials. • Polar coordinates and spherical harmonics: the orbitals of the hydrogen atom. • Cylindrical coordinates and Bessel functions: vibration of a membrane and the bi-harmonic Stokes problem. • Further applications (or functions) can be selected based on the audience (possible topics in fluid mechanics, potential theory, stochastic analysis, numerical solution of PDEs).
Learning goals	Students will learn how to analytically solve some partial differential equations of relevance in applied mathematics. In the process, they will understand the origin and properties of several families of special functions and will appreciate their usefulness in applied mathematics contexts
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic notions of analysis and algebra, ordinary differential equations and partial differential equations
Examination methods (in applicable)	Oral examination on the program and on a student's project
Suggested readings	1. N. N. Lebedev, Special Functions and Their Applications, Prentice–Hall, 1965. 2. G. Arfken, Mathematical Methods for Physicists, 3rd ed., Academic Press, 1985. 3. I. S. Gradshteyn, I. M. Ryzhik, Table of Integrals, Series, and Products, 7th ed., Elsevier, 2007
Additional information	

Course unit English denomination	**Hyperbolic Geometry, Continued Fractions and Cluster Algebras**
Teacher in charge (if defined)	Daniel Labardini Fragoso Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova Email: daniel.labardinifragoso@unipd.it
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	Second semester
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☒ Yes (75% minimum of presence) ☐ No
Course unit contents	1. Hyperbolic Geometry • Möbius transformations • Models and isometries of the hyperbolic plane • The Farey diagram and its symmetries • Ptolemy's identity in the hyperbolic plane 2. Continued Fractions • Recursive formalism • Finite continued fractions • Infinite continued fractions • Examples: the continued fractions of 1+\sqrt{2}, (1+\sqrt{5})/2 and *e* • Equivalent numbers • Continued fractions vs. perfect matchings of snake graphs 3. Cluster Algebras • Basic definitions and examples • The Laurent phenomenon • Cluster variables vs. perfect matchings of snake graphs
Learning goals	To arrive at the notion of Cluster Algebra from two distinct, albeit related, elementary starting points: Hyperbolic Geometry and Continued Fractions
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No

Available for PhD students from other courses	☐ Yes ☒ No
Prerequisites (not mandatory)	The prerequisites are reduced to the minimum: differential and integral calculus of one and several variables; elementary linear algebra; elementary algebra. Knowledge of Differential or Riemannian Geometry would be helpful, but not necessary
Examination methods (in applicable)	According with the teacher
Suggested readings	1. James Anderson. Hyperbolic geometry. Springer-Verlag. Springer Undergraduate Mathematics Series. 2007. 2. Ilke Canakci, Ralf Schiffler. Cluster algebras and continued fractions. Compositio Mathematica 154 (2018), no. 3, 565–593. arXiv:1608.06568 3. Ilke Canakci, Ralf Schiffler. Snake graphs and continued fractions. European Journal of Combinatorics 86 (2020), 103081, 19 pp. arXiv:1711.02461 4. James W. Cannon, William J. Floyd, Richard Kenyon, Walter R. Parry. Hyperbolic Geometry. Flavors of Geometry (edited by Silvio Levy). Cambridge University Press, MSRI Publications, Volume 31. 1997. 5. Sergey Fomin, Andrei Zelevinsky. Cluster algebras IV: Coefficients. Compos. Math. 143 (2007), no. 1, 112–164. arXiv:math/0602259 6. Allen Hatcher. Topology of numbers. American Mathematical Society, 2022. 7. Serge Lang. Introduction to Diophantine approximations, 2nd ed. Springer-Verlag, 1995. 8. William J. LeVeque. Elementary Theory of Numbers. Dover Publications, 1990. 9. Toshihiro Nakanishi. An application of Penner's coordinates of Teichmüller space of punctured surfaces. RIMS Kokyuroku Bessatsu, Kyoto, 2010. B17: Infinite dimensional Teichmller spaces and moduli spaces, 105–114. 10. Robert Penner. Lambda lengths. http://www.ctqm.au.dk/research/MCS/lambdalengths.pdf 11. Robert Penner. Decorated Teichmüller Theory. European Mathematical Society, the QGM Master Class Series. 2012. DOI 10.4171/075 12. Boris Springborn. The hyperbolic geometry of Markov's theorem on Diophantine approximation and quadratic forms. L'Enseignement Mathématique (2) 63 (2017), 333–373. arXiv:1702.05061 13. Lauren K. Williams. Cluster algebras: an introduction. Bull. Amer. Math. Soc. (N.S.) 51 (2014), no. 1, 1–26. arXiv:1212.6263 14. P.M.H. Wilson. Curved spaces, from Classical Geometries to Elementary Differential Geometry. Cambridge University Press. 2008.
Additional information	

Course unit English denomination	**Mathematical Climate Finance**
Teacher in charge (if defined)	Macrina Andrea
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	February 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	1. The carbon equivalence principle and the design of a probability space for climate policymaking 2. Project finance under climate transition risk 3. Internalisation of Scope 3 emissions in the trading book 4. Climate-contingent convertible bonds 5. Stochastic integrated assessment models
Learning goals	This is an introduction to mathematical climate finance by in-depth treatment of some of the recent advances in this burgeoning field of research in financial mathematics. The focus will be on climate transition risk that, together with physical risk, is a major source of climate change risk impacting economies and financial markets. The mathematics, especially the formulation and modelling aspects, on which climate finance is based, is at the core of this course. The aim is thus the study and development of climate finance anchored in financial mathematics. Mathematical climate finance is an active area of research, and this course aims at keeping upto-date with new insights and ongoing research progress in academia and the industry. A taste of the topic treated in this course can be sampled from the cover story in the Fields Notes, Vol. 12:4, Spring/Summer 2024.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Probability and stochastic process theory in continuous time, and MSclevel financial mathematics knowledge/skills.

Examination methods (in applicable)	None
Suggested readings	1. D. Brigo & F. Mercurio (2006) Interest Rate Models – Theory and Practice. Springer Berlin, Heidelberg 2. C. Cormack & A. Macrina (2024) Climate Transition Mitigation: Introducing the CLoCo Bond. SSRN:4851975. 3. C. Cormack & A. Macrina (2024) Sovereign climate-contingent convertible bonds (SCloCo). SSRN:4959445. 4. C. Kenyon, M. Berrahoui & A. Macrina (2024) The Carbon Equivalence Principle: Minimising the Cost to Carbon Net Zero. Risk, Cutting Edge: Climate Finance, Risk.net Feb. 2024. Full version at SSRN:3979608. 5. C. Kenyon, A. Macrina & M. Berrahoui (2023) The Carbon Equivalence Principle: Methodsfor Project Finance. Risk, Cutting Edge: Climate Finance, Risk.net May 2023. Full version at SSRN:4035833. 6. C. Kenyon, A. Macrina & M. Berrahoui (2023) CO2eVA: Pricing the Transition of Scope Emissions. Risk, Cutting Edge: Climate Finance, Risk.net Oct. 2023. Full version at SSRN:4136710. 7. F. Krach, A. Macrina, A. Kanter, E. Hampwaye, S. Hlalukana & N. T. Rateele (2024) The Financial Impact of Carbon Emissions on Power Utilities Under Climate Scenarios. International Journal of Theoretical and Applied Finance, 2450013 (open access). DOI: 10.1142/S0219024924500134 8. A. Macrina (2024) Mathematical climate finance: an investment in our future. Fields Notes, Vol. 12:4, Spring/Summer 2024. 9. G. Kassis & A. Macrina (2024) Arcade processes for informed martingale interpolation. ArXiv:2301.05936 10. S. E. Shreve (2004) Stochastic Calculus for Finance II: Continuous-Time Models. Springer Science & Business Media, LLC.
Additional information	

Course unit English denomination	**Bessel, Cox-Ingersoll-Ross, Ornstein-Uhlenbeck and Gaussian-Volterra processes with Wiener and fractional drivers**
Teacher in charge (if defined)	Mishura Yuliya
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Standard Cox-Ingersoll-Ross and Bessel processes: local and global behaviour of the trajectories as the functionals of coefficients. Fractional Cox-Ingersoll-Ross and Bessel processes and their main properties in dependence of the value of Hurst index. Drift parameters estimation in the standard and fractional Cox-Ingersoll-Ross models. Exact and approximate option pricing under stochastic volatility modeled by fractional Ornstein-Uhlenbeck process. Functional limit theorems for financial markets driven by fractional long-range dependent processes. Fractional Gaussian noise: entropy and alternative entropy functionals, analytical and computational problems related to predictors. Gaussian-Volterra processes as the generalization of fractional Brownian motion. Tempered fractional processes.
Learning goals	To introduce PhD students to the most interesting and modern models of stochastic processes, which, firstly, have various and quite deep analytical, wise-trajectory and asymptotic properties, and secondly, serve as adequate models in financial mathematics, physics, cellular communications, biology, etc. We will consider stochastic differential equations with both the Wiener process and the fractional Brownian process and its generalizations.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	A previous knowledge of the basic concepts of stochastic processes is required. Knowledge of stochastic calculus could help for the advanced

	parts of the course, but during the course the basic concepts will be introduced for the understanding.
Examination methods (in applicable)	Seminar
Suggested readings	1. N. Ikeda, S. Watanabe. Stochastic Differential Equations and Diffusion Processes 2nd Edition - March 1, 1992 2. Cherny, A. S. and Engelbert, H.-J. (2005). Singular Stochastic Differential Equations. Springer Berlin Heidelberg, Berlin, Heidelberg. 3. Lecture Notes on the Yamada-Watanabe Condition for the Pathwise Uniqueness of Solutions of Certain Stochastic Differential Equations S. Altay and Uwe Schmock. 4. Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). A theory of the term structure of interest rates. Econometrica: journal of the Econometric Society, 53(2):385 5. Mishura, Y., Pilipenko, A., and Yurchenko-Tytarenko, A. (2023). Low-dimensional Cox- Ingersoll-Ross process. Stochastics, 2024 arXiv:2303.12911 [math.PR]. 6. Mishura, Y., and Yurchenko-Tytarenko, A. (2023). Standard and fractional reflected Ornstein-Uhlenbeck processes as the limits of square roots of Cox-Ingersoll-Ross processes. Stochastics, 95(1):99-117.
Additional information	

UNIVERSITÀ DEGLI STUDI DI PADOVA

SCIENZE MATEMATICHE

Course unit English denomination	**Numerical cubature and its applications**
Teacher in charge (if defined)	Alvise Sommariva Dipartimento di Matematica ”Tullio Levi-Civita”, Università di Padova Email: alvise@math.unipd.it
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	November 2024
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Quadrature rules: from Newton-Cotes formula to Gaussian rules • Cubature rules on multivariate domains • Quasi-Montecarlo methods • Cubature rules compression by Tchakaloff theorem (time permitting) • Hyperinterpolation on general domain (time permitting) • Numerical solution of Fredholm integral equations of second kind on multivariate domains (time permitting)
Learning goals	The aim of this course is to provide an overview of numerical cubature over multivariate domains, even with complex geometries. Next we show its application to the approximation of continuous functions and to the solution of integral equations
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☒ Yes ☐ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Basic knowledge of analysis, linear algebra and numerical analysis
Examination methods (in applicable)	Oral examination
Suggested readings	1. K. Atkinson, Numerical integration on the sphere, J. Austral. Math. Soc. Ser. B 23 (1981/82), pp. 332–347.

2. K. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press; 1997.
3. J. Dick, F. Kuo, I.H. Sloan, High-dimensional integration: The quasi-Monte Carlo way, Acta Numerica, Cambridge University Press, 2013.
4. C.L. Lawson, R.J. Hanson, Solving Least Squares Problems, Classics in Applied Mathematics 15, SIAM, Philadelphia, 1995.
5. I.H. Sloan, Interpolation and hyperinterpolation over general regions, J. Approx. Theory 83 (1995), pp. 238–254.
6. A. Sommariva, Some cubature rules in Matlab, https://www.math.unipd.it/˜alvise/sets.html.
7. A. Sommariva and M. Vianello, Compression of multivariate discrete measures and applications, Numer. Funct. Anal. Optim. 20 (2015), pp. 1198–1223.

Additional information

Course unit English denomination	**Topics of Control Theory from a Differential Geometric point of view**
Teacher in charge (if defined)	Andrea Spiro, Marta Zoppello
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	• Preliminaries (I). A quick overview of some basic notions and facts of Differential Geometry, as e.g. manifolds, submanifolds, vector fields, push-forwards under diffeomorphisms, tensor fields, p-forms and their integrations on submanifolds, flows, Lie derivatives and Lie brackets, etc. • Preliminaries (II). A brief introduction to a few topics of Control Theory: first order control systems in normal forms, controllability and accessibility, the three variants (Mayer, Lagrange and Bolza) of cost minimising problems under first order differential constraints, Kalman Theorem, needle variations, Pontryagin Maximum Principle (PMP). • Comparison between the classical and the differential geometric proofs of the PMP for Mayer problems. • Frobenius Theorem and its applications to the integrability problems of systems of partial differential equations. • Non-integrable distributions, Rashevskij's Theorem and its generalisations by Chow and Sussmann. • Chow's criterion for the controllability of systems which are linear in controls; Controllability of systems on Lie groups; Discussions of several examples of control systems and comparisons of criterions for local controllabliity. The Chaplygin sleigh and its fluidodynamic variant: results and open problems. • Non-linear control systems and representation of solutions as oriented curves in the extended state-control space. Graphic completions. New criterions for local accessibility and small time local controllability. Examples.
Learning goals	We would like to offer an introduction to a few fundamental results of Control Theory and Differential Geometry, with the purpose of illustrating how certain classical Differential Geometric tools can be used to find solutions to problems in Control Theory. We also have the intent of discussing some

	very recent results by Cardin, Giannotti, Spiro and Zoppello on the Pontryagin Maximum Principle and the local controllability problem of systems with non-linear controls.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Even if the course concerns topics in Control Theory and Differential Geometry, the lectures will be designed to be accessible by a general audience, with no special training in either of those two areas
Examination methods (in applicable)	Oral exam
Suggested readings	1. A. Bressan and B. Piccoli, Introduction to the mathematical theory of control, American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2007. 2. F. Cardin and A. Spiro, Pontryagin maximum principle and Stokes theorem, J. Geom. Phys. 142, (2019), 274–286. 3. F. Cardin, C. Giannotti and Spiro, Andrea, On the Pontryagin maximum principle under differential constraints of higher order, Ann. Polon. Math. 130 (2023), 97–147. 4. W.-L. Chow, U¨ ber Systeme von linearen partiellen Differentialgleichungen erster Ordnung, Math. Ann. 117 (1939), 98–105. 5. J. M. Coron, Control and nonlinearity, American Mathematical Society, Providence, RI, 2007. 6. F. Rampazzo, Lecture notes on Control, Set Separation, and Minima (Optimal control and controllability), (unpublished lectures notes), 2018. 7. P.K. Rashevskij, About connecting two points of complete non-holonomic space by admissible curve (in Russian), Uch. Zapiski Ped. Inst. K. 2 (1938), 83–94. 8. N. Sansonetto and M. Zoppello, On the trajectory generation of the hydrodynamic Chaplygin sleigh, IEEE Control Syst. Lett. 4 (2020), 922–927. 9. S. Sternberg, Lectures on differential geometry, Chelsea Publishing Co., New York, 1983. 10. H. J. Sussmann, Orbits of families of vector fields and integrability of distributions, Trans.Amer. Math. Soc. 180 (1973), 171–188. 11. F. W. Warner, Foundations of Differentiable Manifolds and Lie groups, Springer-Verlag, New York, 1983.
Additional information	

Course unit English denomination	**Introduction to moduli spaces**
Teacher in charge (if defined)	Orsola Tommasi Dipartimento di Matematica ”Tullio Levi-Civita”, Università di Padova Email: orsola.tommasi@unipd.it
Teaching Hours	24
Number of ECTS credits allocated	4
Course period	February-March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Sì (% minima di presenza) ☒ No
Course unit contents	A moduli space is a space parametrizing all possible objects of a certain fixed type. A classical example is the following: let us fix a compact oriented surface ∑. Then the space of all possible complex structures on ∑ is the moduli space M_g of genus *g* Riemann surfaces, where *g* is the topological genus of ∑. By construction, the points of M_g correspond to the isomorphism classes of Riemann surfaces of genus *g*. This kind of construction generalizes to many other classification problems. In good cases, moduli spaces will turn out to be complex manifolds or varieties. However, in most cases the moduli space will not be truly a manifold, but rather a mild generalization of it, called an orbifold. Although the construction of moduli spaces originates in algebraic geometry, moduli spaces themselves are of interest also in other areas of mathematics, such as other areas of geometry, topology, group theory, analysis and mathematical physics. In this course, I would like to present the basic ideas and formalism underlying the concept of moduli space, along with some main examples of interdisciplinary interest. Besides the moduli space of Riemann surfaces, interesting examples with applications in different fields include: • mirror symmetry, a construction from theoretical physics that predicts that there exist pairs of topological spaces (*X,X**) such that the moduli space of complex structures on *X* is isomorphic to the moduli space of symplectic structures on *X**, and vice versa; • modular curves in number theory, which are moduli spaces parametrizing elliptic curves with additional structures.
Learning goals	Learning the fundamentals of moduli spaces
Teaching methods	Lectures

Course on transversal, interdisciplinary, transdisciplinary skills	☒ Yes ☐ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites *(not mandatory)*	Basic notions of geometry, as provided by a Mathematics degree. This course is open to any PhD student. Depending on the students' background, I will include a review of useful notions from geometry and topology.
Examination methods *(if applicable)*	The examination will consist in preparing and giving a talk on a topic connected to moduli spaces
Suggested readings	• Kock, Joachim; Vainsencher, Israel. An invitation to quantum cohomology. Kontsevich's formula for rational plane curves. Progress in Mathematics, 249. Birkh¨auser Boston, Inc., Boston, MA, 2007. xiv+159 pp. • Newstead, P. E. Introduction to moduli problems and orbit spaces. Tata Institute of Fundamental Research Lectures on Mathematics and Physics, 51. Tata Institute of Fundamental • Research, Bombay; by the Narosa Publishing House, New Delhi, 1978. vi+183 pp. • Cox, David A.; Katz, Sheldon. Mirror symmetry and algebraic geometry. Mathematical Surveys and Monographs, 68. American Mathematical Society, Providence, RI, 1999. xxii+469 pp. • Harris, Joe; Morrison, Ian. Moduli of curves. Graduate Texts in Mathematics, 187. Springer-Verlag, New York, 1998. xiv+366 pp.
Additional information	

Course unit English denomination	**Polyhedral structures in algebraic geometry**
Teacher in charge (if defined)	Stefano Urbinati
Teaching Hours	16
Number of ECTS credits allocated	3
Course period	March 2025
Course delivery method	☒ In presence ☐ Remotely ☐ Blended
Language of instruction	English
Mandatory attendance	☐ Yes (% minimum of presence) ☒ No
Course unit contents	Algebraic geometry studies the zero locus of polynomial equations connecting the related algebraic and geometrical structures. In several cases, nevertheless the theory is extremely precise and elegant, it is hard to read in a simple way the information behind such structures. A possible way of avoiding this problem is that of associating to polynomials some polyhedral structures that immediately give some of the information connected to the zero locus of the polynomial. In relation to this strategy I will introduce Newton-Okounkov bodies and Tropical Geometry, underlying the connection between the two theories.
Learning goals	Understand how some deep geometric problem can be translated by a simple polyhedral object.
Teaching methods	Frontal lectures
Course on transversal, interdisciplinary, transdisciplinary skills	☐ Yes ☒ No
Available for PhD students from other courses	☒ Yes ☐ No
Prerequisites (not mandatory)	Good knowledge of commutative algebra, projective geometry and algebraic geometry
Examination methods (in applicable)	Presentation of a part of a research paper
Suggested readings	KL Küronya, A., Lozovanu, V., Local positivity of linear series, arXiv: 1411.6205v1 (2014) PAGI Lazarsfeld, Robert, Positivity in algebraic geometry. I, Classical setting: line bundles and linear series, Springer-Verlag, Berlin, 2004.

PAGII Lazarsfeld, Robert, Positivity in algebraic geometry. II, Positivity for vector bundles, and multiplier ideals, Springer-Verlag, Berlin, 2004.
LM Lazarsfeld, R., Mustaţă, M., Convex bodies associated to linear series, Ann. Sci. Ec.Norm. Supér. (4) 42 (2009), no. 5, 783-835.

Additional information